22107405

International Baccalaureate®
Baccalauréat International
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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

0	0								

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0117

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1.   José stands 1.38 kilometres from a vertical cliff.

   (a)   Express this distance in metres.   *[1 mark]*

   José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff.

diagram not to scale

28.3°

1.38 km
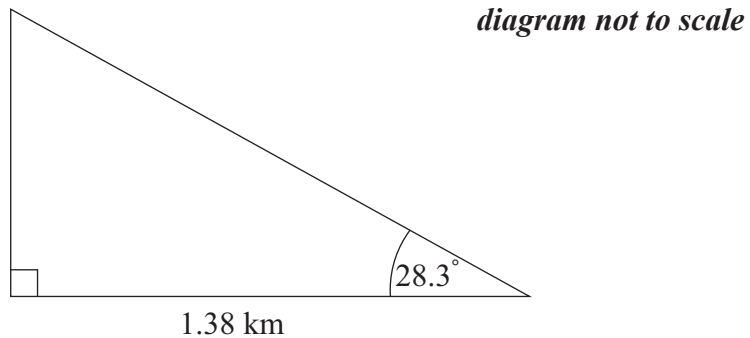
   (b)   Find the height of the cliff according to José's calculation. **Express your answer in metres, to the nearest whole metre.**   *[3 marks]*

   (c)   The actual height of the cliff is 718 metres. Calculate the percentage error made by José when calculating the height of the cliff.   *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

2. (a) Complete the truth table shown below. *[3 marks]*

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$(p \vee (p \wedge q)) \Rightarrow p$
T	T			
T	F			
F	T			
F	F			

(b) State whether the compound proposition $(p \vee (p \wedge q)) \Rightarrow p$ is a contradiction, a tautology or neither. *[1 mark]*

Consider the following propositions.

> *p: Feng finishes his homework*
> *q: Feng goes to the football match*

(c) Write in symbolic form the following proposition.

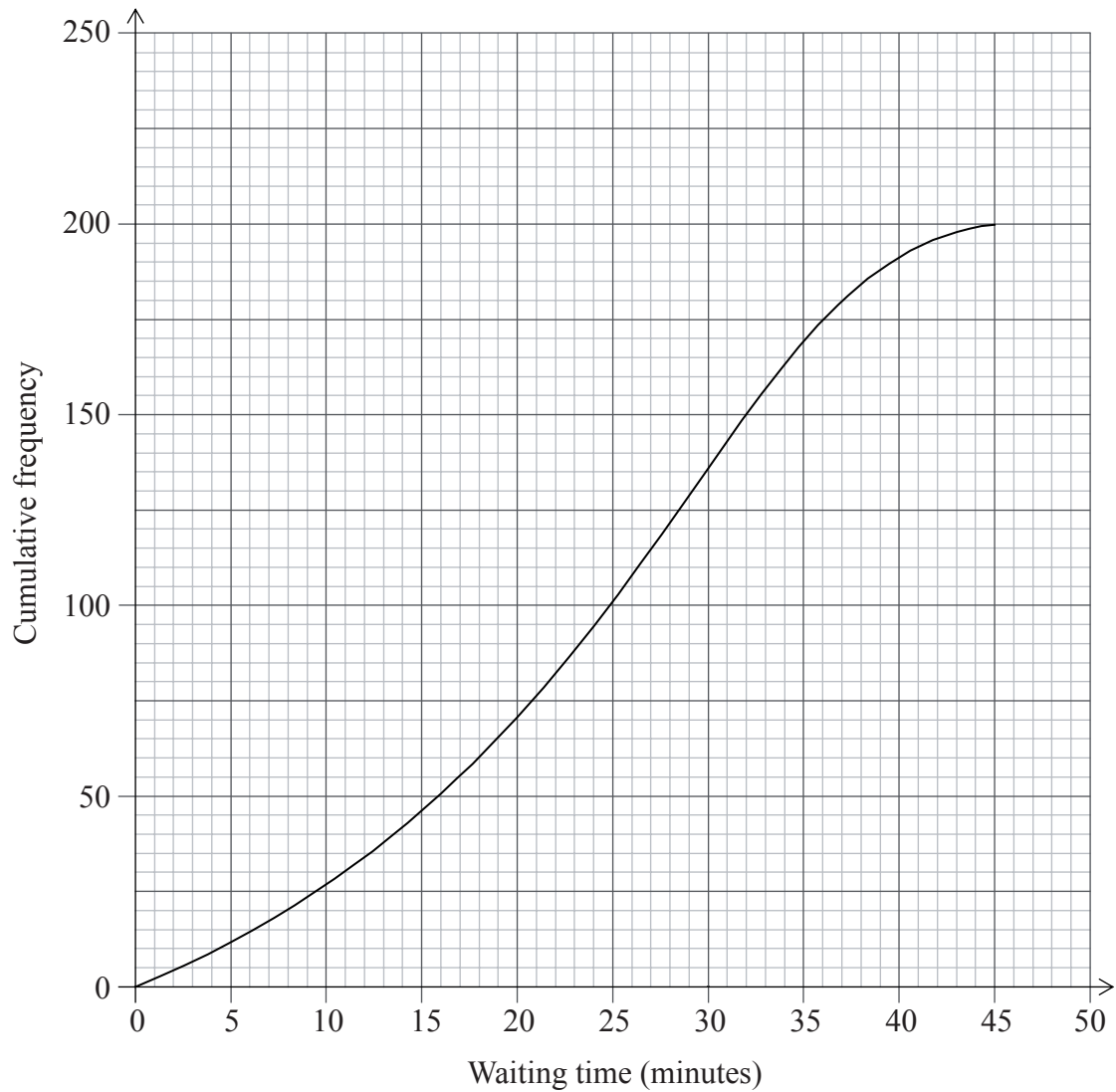> *If Feng does not go to the football match then Feng finishes his homework.* *[2 marks]*

Working:

Answers:

(b) ....

(c) ....

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3. The cumulative frequency graph shows the amount of time in minutes, 200 students spend waiting for their train on a particular morning.

(a) Write down the median waiting time. *[1 mark]*

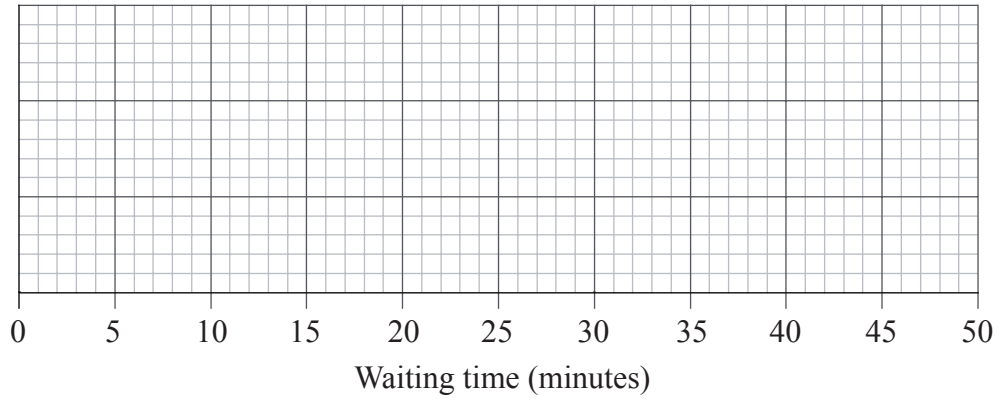(b) Find the interquartile range for the waiting time. *[2 marks]*

(This question continues on the following page)

(Question 3 continued)

The minimum waiting time is zero and the maximum waiting time is 45 minutes.

(c)   Draw a box and whisker plot on the grid below to represent this information.   *[3 marks]*

Waiting time (minutes)

Working:

Answers:

(a) _____

(b) _____

4. The straight line, L_1, has equation $y = -\dfrac{1}{2}x - 2$.

(a) Write down the y intercept of L_1.   *[1 mark]*

(b) Write down the gradient of L_1.   *[1 mark]*

The line L_2 is perpendicular to L_1 and passes through the point $(3, 7)$.

(c) Write down the gradient of the line L_2.   *[1 mark]*

(d) Find the equation of L_2. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.   *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

5. The mean of the ten numbers listed below is 6.8.

$$8, 5, 5, 10, 8, 4, 9, 7, p, q$$

(a) Write down an equation in terms of p and q. *[2 marks]*

The mode of these ten numbers is five and p is less than q.

(b) Write down the value of

    (i) p;

    (ii) q. *[2 marks]*

(c) Find the median of the ten numbers. *[2 marks]*

Working:

Answers:

(a) _____

(b) (i) _____

    (ii) _____

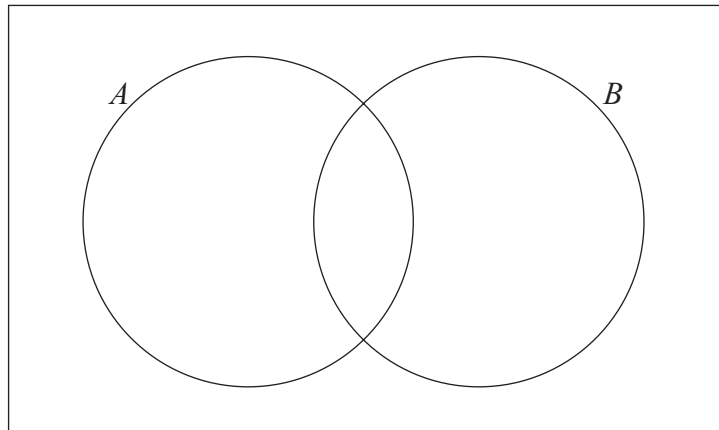(c) _____

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6.   A group of 30 students were asked about their favourite topping for toast.

18 liked peanut butter (A)
10 liked jam (B)
6 liked neither

(a)   Show this information on the Venn diagram below.   *[2 marks]*

(b)   Find the number of students who like both peanut butter and jam.   *[2 marks]*

(c)   Find the probability that a randomly chosen student from the group likes peanut butter, given that they like jam.   *[2 marks]*

Working:

Answers:

(b) _____

(c) _____

7. A concert choir is arranged, per row, according to an arithmetic sequence. There are 20 singers in the fourth row and 32 singers in the eighth row.

(a) Find the common difference of this arithmetic sequence. *[3 marks]*

There are 10 rows in the choir and 11 singers in the first row.

(b) Find the **total** number of singers in the choir. *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

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Turn over

8. Let $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$. *[2 marks]*

(b) Find $P(A \mid B)$. *[2 marks]*

(c) Decide whether A and B are independent events. Give a reason for your answer. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

　　_____

9. A market researcher consulted males and females to determine whether the type of coffee they drink is associated with gender. The types of coffee are Cappuccino, Latte, Americano, Macchiato and Espresso. A χ^2 test was conducted, at the 5 % significance level and the χ^2 value was found to be 8.73.

(a) Write down

    (i) the null hypothesis;

    (ii) the alternative hypothesis. *[2 marks]*

(b) Write down the number of degrees of freedom for this test. *[1 mark]*

(c) Write down the critical value for this test. *[1 mark]*

(d) State whether the type of coffee drunk is independent of gender. Give a reason for your answer. *[2 marks]*

Working:

Answers:

(a) (i) _____

    (ii) _____

(b) _____

(c) _____

(d) _____

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10. Astrid invests 1200 Euros for five years at a nominal annual interest rate of 7.2 %, **compounded monthly**.

(a) Find the interest Astrid has earned during the five years of her investment. **Give your answer correct to two decimal places**. *[3 marks]*

Helen invests 1200 Euros in an annual **simple interest** scheme for five years. She earns **the same** interest as Astrid.

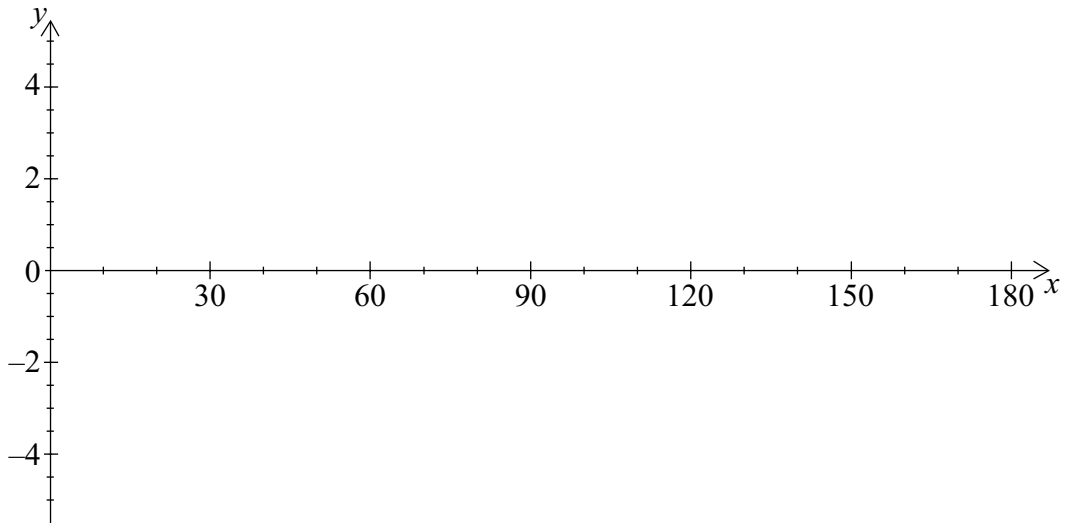(b) Find the simple interest rate of this scheme. *[3 marks]*

Working:

Answers:

(a) _____

(b) _____

11. Consider the function $y = 3\cos(2x) + 1$.

(a)   Sketch the graph of this function for $0 \le x \le 180°$.   *[3 marks]*

(b)   Write down the period of the function.   *[1 mark]*

(c)   Using your graphic display calculator find the **smallest** possible value of x, $0 \le x \le 180°$, for which $3\cos(2x) + 1 = 2$.   *[2 marks]*

Working:

Answers:

(b) _____

(c) _____

12. A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it is first started.

(a)   Find the number of teenagers who started the rumour.   *[2 marks]*

(b)   Write down the number of teenagers who have heard the rumour five hours after it is first started.   *[1 mark]*

(c)   Determine the length of time it would take for 150 teenagers to have heard the rumour. **Give your answer correct to the nearest minute.**   *[3 marks]*
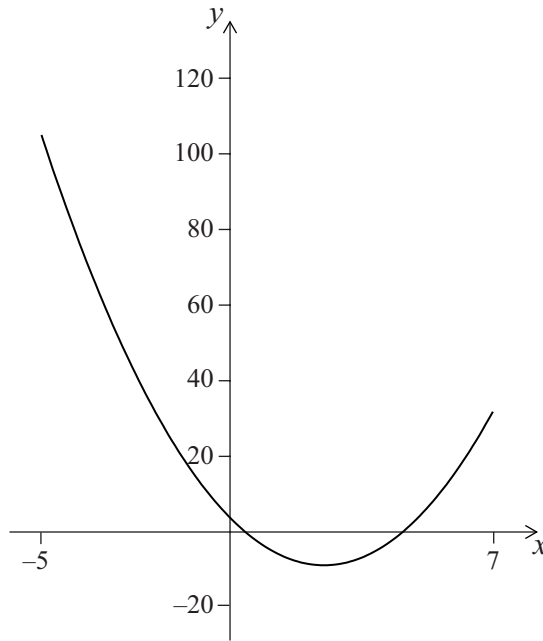
Working:

Answers:

(a)   _____

(b)   _____

(c)   _____

13. The graph of $y = 2x^2 - rx + q$ is shown for $-5 \le x \le 7$.

The graph cuts the y axis at $(0, 4)$.

(a) Write down the value of q. *[1 mark]*

The axis of symmetry is $x = 2.5$.

(b) Find the value of r. *[2 marks]*

(c) Write down the minimum value of y. *[1 mark]*

(d) Write down the range of y. *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

(d) _____

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14. A rectangular cuboid has the following dimensions.

Length   0.80 metres   (AD)
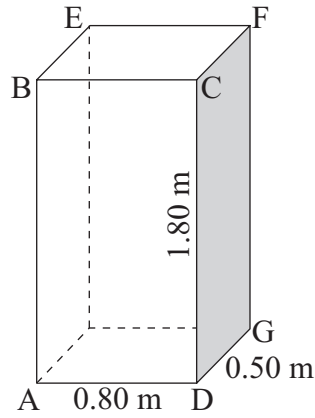Width   0.50 metres   (DG)
Height   1.80 metres   (DC)

diagram not to scale

(a)   Calculate the length of AG.   *[2 marks]*

(b)   Calculate the length of AF.   *[2 marks]*

(c)   Find the size of the angle between AF and AG.   *[2 marks]*

Working:

Answers:

(a) _____

(b) _____

(c) _____

15. The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

(a)   State whether $f(0)$ is greater than, less than or equal to $f(-2)$. Give a reason for your answer.   *[2 marks]*

The point $P(-2, 3)$ lies on the graph of $f(x)$.

(b)   Write down the equation of the tangent to the graph of $f(x)$ at the point P.   *[2 marks]*

(c)   From the information given about $f'(x)$, state whether the point $(-2, 3)$ is a maximum, a minimum or neither. Give a reason for your answer.   *[2 marks]*

Working:

Answers:

(a) _____

_____

(b) _____

(c) _____

_____